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Large-cell model of radiation heat transfer in multiphase flows typical for fuel–coolant interaction

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Abstract

The visible and near-infrared radiative properties of water containing polydisperse steam bubbles and core melt particles are analyzed. The alternative approximate models of radiation heat transfer in the range of water semi-transparency are considered. A new approach based on radiation balance equations for a large computational cell of size about several centimeters is suggested. This approach called large-cell model takes into account not only water heating but also radiation heat transfer between the particles of different temperatures. The latter is important for problems of fuel–coolant interaction in possible severe accident of light water nuclear reactors. The model problems considered in the paper give estimates of nonlocal effects of thermal radiation and confirm the applicability of the large-cell model at parameters typical for fuel–coolant interaction.

Keywords: Thermal radiation; Multiphase flow; Semi-transparent; Particle; Bubble; Fuel–coolant interaction

1. Introduction

The so-called fuel–coolant interaction (FCI) problem has been widely investigated during last two decades because of possible severe accident of light-water nuclear reactors. The complexity of different stages of interaction of high-temperature core melt with water is one of the reasons of the present-day state of the art when some important physical processes have not been considered in details yet [1]. The efforts of many researchers have been focused on hydrodynamic simulation of melt jet breakup [2–4] and specific problems of steam explosion [5–8]. At the same time, the radiation heat transfer in the multiphase medium containing polydisperse corium particles of temperature of about 2500–3000 K was not a subject of detailed analysis. The paper by Dinh et al. [9] was probably the first publication where the important role of radiation heat transfer has been discussed. It was noted that a part of thermal radiation emitted by particles can be absorbed far from the radiation sources because of semi-transparency of water in the short-wave range.

The general problem of radiation heat transfer between corium particles and ambient water can be separated to the following problems of different scale: thermal radiation from a single particle through a steam mantle to ambient water and the radiation heat transfer in a large-scale volume containing numerous corium particles and steam bubbles. One can show that solutions to these problems can be incorporated in a general physical and computational model as it was done for similar problems of radiation heat transfer in other disperse systems [10]. The single-particle problem has been analyzed in details in papers [11–13], where the main attention was paid to significant contribution of electromagnetic wave effects in the case of very thin steam layer. The effect of semi-transparency of nonisothermal oxide particles on their thermal radiation has been studied in papers [14–16]. The resulting physical pictures of particle solidification have been analyzed in recent papers [17,18]. To the best of the author's knowledge, the first attempt to calculate radiation heat transfer in water

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Nomenclature

a	particle radius
B_λ	Planck's function
c	heat capacity
c_0	velocity of light
D	radiation diffusion coefficient
f_v	volume fraction
F	particle size distribution
h	heat transfer coefficient
I_λ	spectral radiation intensity
l	penetration depth of radiation
n	index of refraction
q	radiative flux
p_λ	spectral radiation power
P	integral radiation power
Q_a	efficiency factor of absorption
Q_s^{tr}	transport efficiency factor of scattering
r	radial coordinate
$\vec{r}$	vector coordinate
T	temperature
x	diffraction parameter

Greek symbols

α	absorption coefficient
δ	thickness of steam layer, Dirac function
κ	index of absorption
λ	radiation wavelength
v	relative volume fraction of small particles
ρ	density
σ	scattering coefficient
τ	optical thickness
φ	function defined by Eq. (34)
ξ	relative number of small particles
ψ	coefficient of radiative cooling rate defined by Eq. (30)
ζ, ζ_0	coefficients defined by Eqs. (32) and (33)
$\vec{\Omega}$	unit vector of direction

Subscripts and superscripts

a	absorption
b	bubble
c	corium
e	external
s	scattering, steam
tr	transport
w	water
δ	steam layer
λ	spectral
(1)	semi-transparency range
(2)	opacity range

containing corium particles was made by Yuen [19]. It was assumed that there is no radiation scattering in the medium. The spectral radiative properties of the melt particles of various temperatures and sizes were ignored in this paper and all the particles were considered as the sources of black-body radiation. The calculations of paper [19] were based on the formal zonal method which seems to be not a good choice for the problem considered.

The objective of the present paper is to develop a simple model for radiation heat transfer calculation in water containing numerous polydisperse corium particles of different temperatures and polydisperse steam bubbles. The model should be rather simple to be implemented in problem-oriented CFD-codes for multiphase flow calculations.

The reasonable choice of the radiation model should be based on spectral properties of water, steam bubbles and corium particles in the visible and near infrared spectral ranges. For this reason, we start our study from the analysis of the main radiative characteristics of the multiphase medium typical for FCI problems. We will have also in mind that aluminum oxide (weakly absorbing in the visible and near infrared [10]) can be used in model experiments as a substance simulated corium [1].

It is clear that the simplest approach for the radiation heat transfer should give only some additional relations for heat exchange inside single computational cells. Such radiation-balance approach suggested in the paper is called large-cell model because it resembles a sub-grid scale model in large-eddy simulation of turbulent flows [20].

2. Spectral radiative properties of the medium

Generally speaking, the relation between properties of single particles and the corresponding characteristics of ensembles of particles in a disperse medium is not simple especially in the case of closely spaced or regularly positioned particles [21–24]. In this paper, we assume that all the particles and bubbles are randomly placed in the volume (there is no any regular structure) and the distance between neighboring particles is greater than their sizes. It makes the traditional assumption of independent scattering to be justified: both absorption and scattering of radiation by a single particle are the same as those determined in the absence of all the other particles.

We will not consider the detailed angular characteristics of radiation scattered by particles and bubbles. Our analysis is based on the so-called transport approximation when scattering phase function is presented as a sum of isotropic component and Dirac δ-function in the forward direction [10,25]. The transport approximation, which has been used first in neutron transport theory [26], allows reducing the radiative transfer problem to the form similar to that for isotropic scattering but with transport scattering coefficient σ_λ^{tr} instead of σ_λ. This procedure is known also as the "isotropic scaling" [27,28]. As a result, we have only two spectral characteristics of the medium: absorption coefficient α_λ and transport scattering coefficient σ_λ^{tr}. The experience of heat transfer calculations in disperse systems showed that transport approximation is sufficiently accurate for calculating thermal radiation in many applications [10,29].

To suggest an adequate spectral model of radiation heat transfer, one should take into account specific properties of water in the visible and near infrared spectral ranges.

2.1. Optical constants of water

The optical constants of water in the most important wavelength range from 0.5 to 5 μm are shown in Fig. 1. The values of n_w and κ_w are obtained by linear interpolation of tabulated data from papers [30] (in the range of $0.5 \leqslant \lambda < 2$ μm) and [31] (in the range of $2 \leqslant \lambda \leqslant 5$ μm). One can see in Fig. 1 that water is transparent in a short-wave range and there is a strong absorption band at the wavelength $\lambda = 3$ μm. The spectral absorption coefficient of water can be calculated as follows:

$$\alpha_w = 4\pi\kappa_w/\lambda \quad (1)$$

One can also introduce the characteristic penetration depth of the collimated radiation in water: $l_\lambda = 1/\alpha_w$. The spectral dependence of l_λ in the most interesting intermediate range $0.8 < \lambda < 1.4$ μm is illustrated in Fig. 2. One can see that l_λ decreases from about 0.5 m at the visible range boundary $\lambda = 0.8$ μm to $l_\lambda = 1$ mm at the wavelength $\lambda = 1.38$ μm. It makes reasonable to consider separately the following conventional spectral regions:

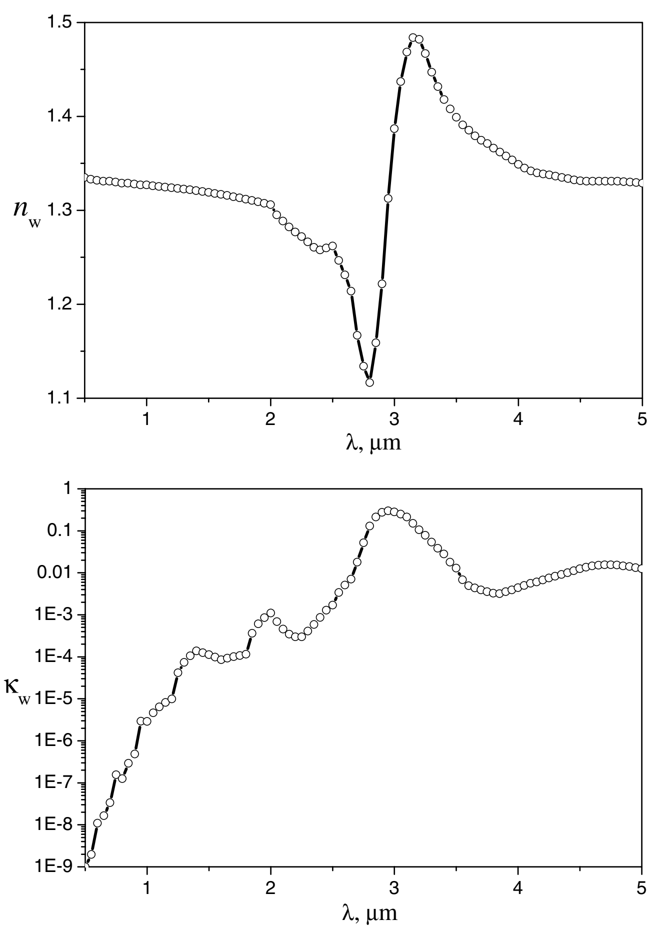

Fig. 1. Optical constants of water in the visible and near infrared.

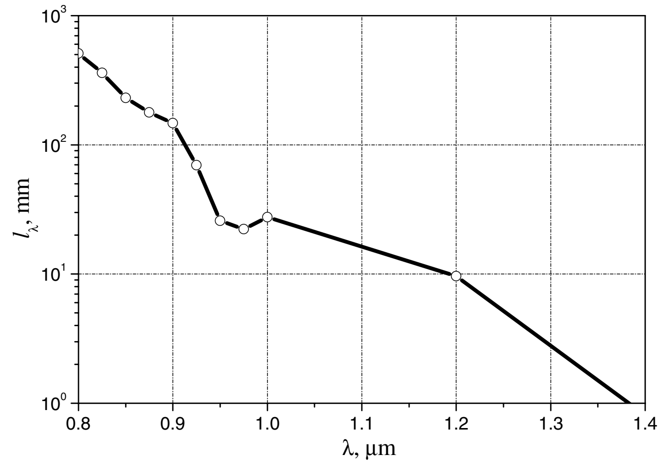

Fig. 2. The characteristic propagation depth of collimated near-infrared radiation in water.

– The short-wave semi-transparency range $\lambda < \lambda_* = 1.2$ μm, where $l_\lambda > 10$ mm. There is a considerable radiation heat transfer between corium particles in this spectral range because the distance between neighboring particles is usually less than 10 mm. One can use the traditional radiation transfer theory to calculate the volume distribution of radiation power. Both absorption and scattering of radiation by particles should be taken into account [10].
– The opacity range $\lambda > \lambda_*$, where $l_\lambda < 10$ mm. In this range, one can neglect the radiation heat transfer between the particles. The radiative transfer problem degenerates because of strong absorption at distances comparable with both particle sizes and distances between the particles. One can assume that radiation emitted by the particle in this spectral range is totally absorbed in ambient water.

Of course, the above two-region scheme should be treated as a simple approach and the effect of conventional value of the boundary wavelength λ^* may be a subject of the further analysis.

In a multiphase flow typical for FCI problem, the numerous steam bubbles and core melt particles have a significant effect on radiative properties of the medium in the range of water semi-transparency.

2.2. Absorption and scattering characteristics of water containing bubbles and particles

It is known that absorption and scattering of radiation by spherical particles of arbitrary size can be calculated by using the rigorous Mie theory [10,32,33]. In our case, all the particles and bubbles are much greater than radiation wavelength [2–4]. We will assume that thickness of concentric steam layer (steam mantle) separating hot particles from ambient water is also much greater than the wavelength [9]. The latter enables us to use the geometrical optics approximation for the analysis of the radiative properties of single particles. Remember that electromagnetic wave effects observed at very small thickness of surface steam layer are considered in details in papers [11–13].

The general expressions for absorption coefficient and transport scattering coefficient of semi-transparent water containing polydisperse spherical bubbles of radius a_b and corium particles of radius a are as follows [10,34]:

$$\alpha_\lambda = \alpha_{\lambda w} + 0.75\frac{f_v^s}{a_{30}^b}\int_0^\infty Q_a^b a_b^2 F_b(a_b)\,da_b + 0.75\frac{f_v^c}{a_{30}}\int_0^\infty Q_a a^2 F(a)\,da \tag{2}$$

$$\sigma_\lambda^{tr} = 0.75\frac{f_v^b}{a_{30}^b}\int_0^\infty Q_s^{b,tr} a_b^2 F_b(a_b)\,da_b + 0.75\frac{f_v^c}{a_{30}}\int_0^\infty Q_s^{tr} a^2 F(a)\,da \tag{3}$$

where f_v^s, f_v^c are the local volume fractions of steam and corium, F_b, F are the size distributions of bubbles and corium particles, Q_a and Q_s^{tr} are the efficiency factor of absorption and the transport efficiency factor of scattering, while the parameter a_{30} can be computed from the following definition of a_{ij}:

$$a_{ij} = \int_0^\infty a^i F(a)\,da \Bigg/ \int_0^\infty a^j F(a)\,da \tag{4}$$

The radiative properties of gas bubbles in an absorbing and refracting medium have been recently studied by the author [34]. The efficiency factor of absorption Q_a^b and the transport efficiency factor of scattering $Q_s^{b,tr}$ for spherical gas bubbles embedded in a refracting and weakly absorbing host medium with optical constants n_e and κ_e have been calculated by using the Mie theory. The calculations were performed in the range of index of refraction $1.2 \leqslant n_e \leqslant 1.5$ for two values of the absorption index: $\kappa_e = 10^{-4}$ and 10^{-3}. It was shown that the efficiency factors of large bubbles can be approximated by the following simple asymptotic relations for diffraction parameter $x_b = 2\pi a_b/\lambda \gg 1$ and optical thickness $\tau_e = 2\kappa_e x_b \ll 1$:

$$Q_a^b = -8\kappa_e x_b/3, \quad Q_s^{b,tr} = 0.9(n_e - 1) \tag{5}$$

Equations (5) overestimate the absolute values of Q_a^b and $Q_s^{b,tr}$ by less than 5% in the range $20 < x_b \ll 1/(2\kappa_e)$. The resulting approximate expressions for the absorption coefficient and transport scattering coefficient of water containing polydisperse steam bubbles are as follows [34]:

$$\alpha_\lambda = (1 - f_v^s)\alpha_w, \quad \sigma_\lambda^{tr} = 0.675(n_w - 1)\frac{f_v^s}{a_{32}^b} \tag{6}$$

where f_v^s is the volume fraction of steam. It is important that absorption does not depend on the bubble size distribution and scattering does not depend on water absorption index. The only parameter related to the bubbles which affects the transport scattering coefficient of the medium is the ratio of volume fraction of steam to the average radius of bubbles: f_v/a_{32}^b. Note that these equations have been verified in paper [35] in the experimental study of radiative properties of fused quartz containing gas bubbles.

Consider now absorption and scattering of radiation by spherical corium particles in water. We assume that every particle is surrounded by a concentric steam layer that separates the particle from ambient water. To make clear the effect of steam layer on absorption and transport scattering efficiency factors, consider some results of Mie theory calculations for not so large particles. We will use the values of optical constants typical for metal oxides ($n = 2$, $0.01 \leqslant \kappa \leqslant 0.5$), the fixed value of diffraction parameter of the particle $x = 2\pi a/\lambda = 50$ and the variable parameter $x_\delta = 2\pi\delta/\lambda$. Following the papers [35–38], the generalized Mie solution for two-layer spheres embedded in refracting nonabsorbing medium has been used in the calculations.

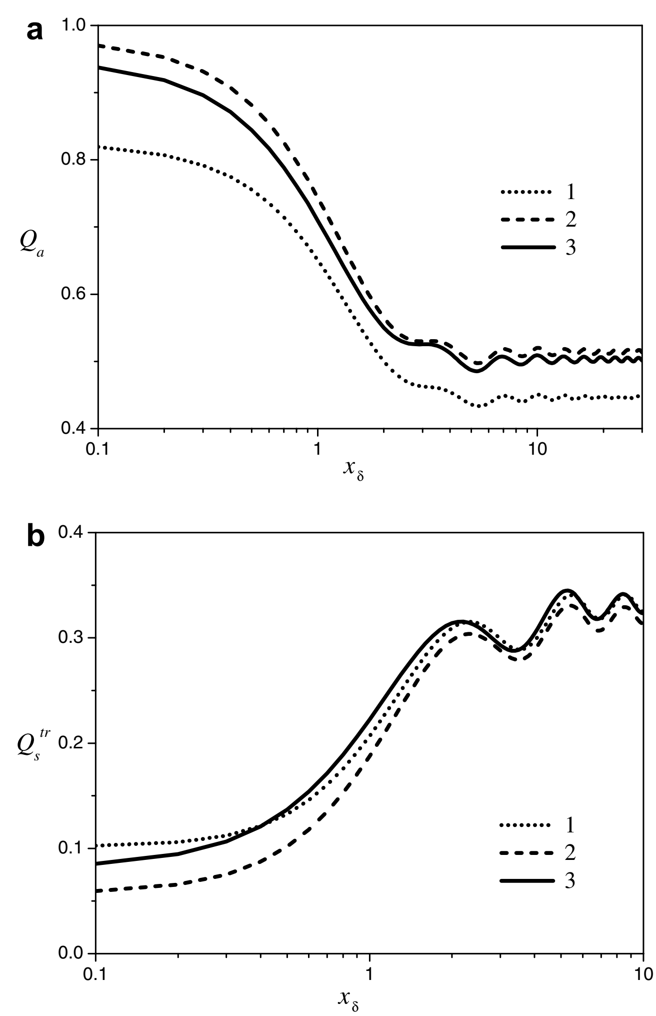

Fig. 3. Effect of steam layer thickness on absorption (a) and scattering (b) of radiation by oxide particle: 1– $\kappa = 0.01$, 2 – $\kappa = 0.1$, 3 – $\kappa = 0.5$. The efficiency factor of absorption Q_a is referred to the particle cross section, but transport efficiency factor of scattering Q_s^{tr} – to the steam bubble cross section.

The index of refraction of water is assumed constant and equal to $n_w = 1.33$. The results of calculations are presented in Fig. 3. One can see that variation of absorption index of particle substance in the range from $\kappa = 0.01$ to 0.5 does not change the character of the curves $Q_a(x_\delta)$ and $Q_s^{tr}(x_\delta)$. The significant change of the steam layer effect is observed in the intermediate range of $0.5 < x_\delta < 2$ with transfer from the wave region $x_\delta \ll 1$ to the region of geometrical optics $x_\delta \gg 1$. The asymptotic value of the absorption efficiency factor referred to the particle cross is as follows:

$$Q_a = Q_a^0/n_w^2 \tag{7}$$

where Q_a^0 is the absorption efficiency factor of the oxide particle in vacuum. Note that this result agrees with the earlier analysis [11–13]. In contrast to absorption, the transport efficiency factor of scattering referred to the steam bubble cross section increases with x_δ from a relatively small value to the asymptotic value of Q_s^{tr} for the steam bubble without oxide particle:

$$Q_s^{tr} = 0.9(n_w - 1) \tag{8}$$

In this paper, the thickness of steam layer δ is assumed much less than the particle radius a. Therefore, the expression (8) can be used for the transport efficiency factor of scattering referred to the particle radius.

To use Eq. (7), one needs the value of absorption efficiency factor of the particle in vacuum Q_a^0. Following papers [17,18] one can assume that millimeter-size particles of corium are opaque both in the visible and near infrared spectral ranges. In this case, $Q_a^0 = \varepsilon_c$, where ε_c is the spectral hemispherical emissivity of bulk corium. The smaller corium particles as well as large particles of core melt simulants (like aluminum oxide) are expected to be semi-transparent [18]. For this reason, we consider the general case of semi-transparent oxide particles. By analogy with approximation suggested in paper [39], one can use the following simple expression:

$$Q_a^0 = \varepsilon_c[1 - \exp(-2\alpha_c a)] \tag{9}$$

where α_c is the spectral absorption coefficient of particle substance. The resulting expression for absorption efficiency factor of the particle in water can be obtained by substitution of Eq. (9) into Eq. (7).

Let us discuss an applicability of monodisperse approximation. Obviously, the absorption coefficient of polydisperse system does not depend on particle size distribution when Q_a is directly proportional to the particle radius a. It is clear also that monodisperse model with average radius a_{32} is adequate in the case of $Q_a = \text{const}$. The calculations showed that the same monodisperse model is rather good for intermediate behavior of $Q_a(a)$ given by exponential expression like Eq. (9) [10,40,41]. For this reason, the monodisperse model with average radius a_{32} is employed in this paper. The resulting equations for absorption and scattering characteristics of semi-transparent water containing steam bubbles and corium particles are as follows:

$$\alpha_\lambda = (1 - f_v^s)\alpha_w + \alpha_c, \quad \alpha_c = 0.75\frac{f_v^c}{a_{32}}\frac{Q_a^0(a_{32})}{n_w^2},$$
$$\sigma_\lambda^{tr} = 0.675(n_w - 1)\left(\frac{f_v^s}{a_{32}^b} + \frac{f_v^c}{a_{32}}\right) \tag{10}$$

2.3. Thermal radiation from hot particles

There are two principal difficulties in determination of thermal radiation of polydisperse particles in FCI:

- The cooling rate of corium particles in water depends strongly on the particle radius. As a result, the neighboring particles of different size may have quite different temperatures.
- One can expect a considerable temperature difference between the center and the surface of large particle because of fast radiative cooling and very low thermal conductivity of metal oxides [17,18].

We are going to take into account the first of these effects in radiation heat transfer modeling. It means that monodisperse model used in the above expression for absorption coefficient of polydisperse corium particles is insufficient for thermal radiation calculations. One should take into account the partial absorption coefficients of all the particle fractions to construct the source term in thermal radiation calculations. As for radiation from nonisothermal particles, it has been studied in a set of previous papers [14–18]. In this paper, the following simple relation for spectral radiative flux from a unit surface of a single particle of corium in water is employed:

$$q_\lambda^1 = Q_a n_w^2 \pi B_\lambda(T) = Q_a^0 \pi B_\lambda(T) \tag{11}$$

Eq. (11) is correct in the case of totally opaque particle when one should put $Q_a^0 = \varepsilon_c$ and consider T as the particle surface temperature. For semi-transparent particles, one should use more general expression (9) for efficiency factor of absorption and the temperature T can be approximately treated as a volume averaged temperature of the particle [17,18]. For simplicity, we will assume that particles of the same size have the same temperature. In this case, thermal radiation power generated by all the particles in a unit volume of the medium can be written as follows:

$$p_\lambda = 3\frac{f_v^c}{a_{30}}\int_0^\infty q_\lambda^1(a)a^2F(a)\,da \tag{12}$$

or

$$p_\lambda = 3\frac{f_v^c}{a_{32}}Q_a^0(a_{32})\pi\overline{B}_\lambda = 4\alpha_c\pi n_w^2\overline{B}_\lambda,$$
$$\overline{B}_\lambda = \frac{\int_0^\infty Q_a^0(a)a^2F(a)B_\lambda[T(a)]\,da}{Q_a^0(a_{32})a_{20}} \tag{13}$$

We have now all the necessary to formulate the radiation heat transfer problem in water containing steam bubbles and hot particles.

3. The radiation heat transfer problem

By using the transport approximation for scattering phase function, the radiative transfer equation (RTE) can be written as follows [10,42]:

$$\vec{\Omega}\nabla I_\lambda(\vec{r},\vec{\Omega}) + \beta_\lambda^{\rm tr}(\vec{r})I_\lambda(\vec{r},\vec{\Omega}) = \frac{\sigma_\lambda^{\rm tr}(\vec{r})}{4\pi}\int_{(4\pi)} I_\lambda(\vec{r},\vec{\Omega}')\,{\rm d}\vec{\Omega}' + \frac{p_\lambda(\vec{r})}{4\pi} \tag{14}$$

where $I_\lambda(\vec{r},\vec{\Omega})$ is the spectral intensity of radiation at point $\vec{r}$ in direction $\vec{\Omega}$, $\beta_\lambda^{\rm tr} = \alpha_\lambda + \sigma_\lambda^{\rm tr}$ is the transport extinction coefficient. Of course, thermal radiation of water is negligible and it is omitted in this equation. Integration of RTE over all values of solid angle gives the radiation balance equation:

$$\nabla\vec{q}_\lambda = p_\lambda(\vec{r}) - \alpha_\lambda(\vec{r})I_\lambda^0(\vec{r}) \tag{15}$$

where $q_\lambda(\vec{r})$ is the local spectral radiative flux, $I_\lambda^0(\vec{r})$ is the value proportional to the local radiation energy density $I_\lambda^0(\vec{r})/c_0$:

$$\vec{q}_\lambda(\vec{r}) = \int_{(4\pi)} I_\lambda(\vec{r},\vec{\Omega})\vec{\Omega}\,{\rm d}\vec{\Omega},\quad I_\lambda^0(\vec{r}) = \int_{(4\pi)} I_\lambda(\vec{r},\vec{\Omega})\,{\rm d}\vec{\Omega} \tag{16}$$

The simplest differential approximations, brought together by the general term "diffusion approximation", give the following representation of the spectral radiative flux [10]:

$$\vec{q}_\lambda = -D_\lambda(\vec{r})\nabla I_\lambda^0 \tag{17}$$

and differ only by expressions for radiation diffusion coefficient D_λ. Sometimes the term "diffusion approximation" is related only to the case when

$$D_\lambda = 1/(3\beta_\lambda^{\rm tr}) \tag{18}$$

that corresponds to Eddington approximation, or P_1 approximation of the method of spherical harmonics [10]. Note that Eqs. (17) and (18) can be derived by assumption of a linear angular dependence of radiation intensity [10,42]:

$$I_\lambda(\vec{r},\vec{\Omega}) = \frac{1}{4\pi}\left[I_\lambda^0(\vec{r}) + 3\vec{\Omega}\vec{q}_\lambda(\vec{r})\right] \tag{19}$$

The latter assumption seems to be rather good for the problem considered. Therefore, we will consider P_1 approximation instead of the RTE. The resulting boundary-value problem can be written as follows:

$$-\nabla(D_\lambda\nabla I_\lambda^0) = p_\lambda(\vec{r}) - \alpha_\lambda I_\lambda^0(\vec{r}) \tag{20}$$

$$-D_\lambda\nabla I_\lambda^0\cdot\vec{n} = I_\lambda^0/2 \quad \text{at the region boundary} \tag{21}$$

The Marshak boundary condition (21) ($\vec{n}$ is the external normal to the boundary surface) is written here for the case of zero external radiation and reflection from the boundary surface [10]. After solving the problem (20) and (21) for several wavelengths in the range of $\lambda_1 < \lambda < \lambda_*$, one can find the radiation power absorbed in water:

$$P_{\rm w}(\vec{r}) = P_{\rm w}^{(1)} + P_{\rm w}^{(2)},$$
$$P_{\rm w}^{(1)}(\vec{r}) = (1-f_{\rm v}^{\rm s})\int_{\lambda_1}^{\lambda_*}\alpha_{\rm w}I_\lambda^0(\vec{r})\,{\rm d}\lambda,\quad P_{\rm w}^{(2)}(\vec{r}) = \int_{\lambda_*}^{\lambda_2} p_\lambda(\vec{r})\,{\rm d}\lambda \tag{22}$$

where the components $P_{\rm w}^{(1)}$, $P_{\rm w}^{(2)}$ correspond to the ranges of water semi-transparency and opacity. One can assume that $P_{\rm w}^{(1)}$ is spent on volume heating of water whereas $P_{\rm w}^{(2)}$ – only for water evaporation near the hot particles. The resulting thermal radiation flux from a unit surface of a single particle can be calculated as follows:

$$q_{\rm c}(\vec{r},a) = q_{\rm c}^{(1)} + q_{\rm c}^{(2)}$$
$$q_{\rm c}^{(1)}(\vec{r},a) = \int_{\lambda_1}^{\lambda_*} Q_{\rm a}^0\left\{\pi B_\lambda[T(\vec{r},a)] - \frac{I_\lambda^0(\vec{r})}{4n_w^2}\right\}{\rm d}\lambda, \tag{23}$$
$$q_{\rm c}^{(2)}(\vec{r},a) = \pi\int_{\lambda_*}^{\lambda_2} Q_{\rm a}^0 B_\lambda[T(\vec{r},a)]\,{\rm d}\lambda$$

where $q_{\rm c}^{(1)}$, $q_{\rm c}^{(2)}$ correspond to the conventional spectral ranges of water semi-transparency and opacity. The total radiative heat loss from corium particles

$$P_{\rm c}(\vec{r}) = P_{\rm c}^{(1)} + P_{\rm w}^{(2)},$$
$$P_{\rm c}^{(1)}(\vec{r}) = \int_{\lambda_1}^{\lambda_*}\{p_\lambda(\vec{r}) - \alpha_{\rm c}(\vec{r})I_\lambda^0(\vec{r})\}\,{\rm d}\lambda \tag{24}$$

Note that $P_{\rm c}^{(1)} \neq P_{\rm w}^{(1)}$ due to heat transfer by radiation in semi-transparent medium:

$$P_{\rm c}^{(1)} - P_{\rm w}^{(1)} = \int_{\lambda_1}^{\lambda_*}[p_\lambda(\vec{r}) - \alpha_\lambda I_\lambda^0(\vec{r})]\,{\rm d}\lambda \tag{25}$$

4. The large-cell model based on balance equations

The complete solution to the radiation heat transfer problem in a multiphase flow typical for fuel–coolant interaction is too complicated even in the case when P_1 approximation is employed. The main computational difficulty is related with wide range of optical thickness of the medium at different wavelengths. One should consider not only the visible radiation when optical thickness of the medium is determined by numerous particles but also the short-wave part of the infrared range characterized by large absorption coefficient of water. As a result, the numerical solution of the boundary-value problem (20) and (21), generally speaking, cannot be performed by using the same computational mesh at all wavelengths.

The idea of the large-cell model is to take into account the usual approach employed in multiphase flow calculations when the computational region is divided into large cells of size about several centimeters with constant characteristics of the medium in single cells. Eq. (20) can be solved analytically in every element because of constant radiative characteristics of the medium. After that, one can consider the coupled algebraic equations for unknown coefficients of boundary equations corresponding to the radiative transfer between the neighboring cells. In this paper, we consider the simplest approach based on the assumption of negligible radiation heat transfer between the cells. An applicability of the latter assumption will be estimated below. The local radiative balance in every cell gives the following relation instead of the problem (20) and (21):

$$I^0_\lambda = p_\lambda/\alpha_\lambda \tag{26}$$

As a result, the general expressions (22)–(24) can be replaced by the following ones:

$$P_\mathrm{w} = P_\mathrm{c} = P^{(1)}_\mathrm{w} + P^{(2)}_\mathrm{w}, \quad P^{(1)}_\mathrm{w} = (1 - f^\mathrm{s}_\mathrm{v}) \int_{\lambda_1}^{\lambda_*} \frac{\alpha_\mathrm{w}}{\alpha_\lambda} p_\lambda \,\mathrm{d}\lambda,$$

$$P^{(2)}_\mathrm{w} = \int_{\lambda_*}^{\lambda_2} p_\lambda \,\mathrm{d}\lambda \tag{27}$$

$$q_\mathrm{c}(a) = q^{(1)}_\mathrm{c} + q^{(2)}_\mathrm{c} \tag{28}$$

$$q^{(1)}_\mathrm{c}(a) = \pi \int_{\lambda_1}^{\lambda_*} Q^0_\mathrm{a} \{B_\lambda[T(a)] - \frac{\alpha_\mathrm{c}}{\alpha_\lambda} \overline{B}_\lambda\} \,\mathrm{d}\lambda,$$

$$q^{(2)}_\mathrm{c}(a) = \pi \int_{\lambda_*}^{\lambda_2} Q^0_\mathrm{a} B_\lambda[T(a)] \,\mathrm{d}\lambda \tag{29}$$

One can see that scattering of radiation by steam bubbles and steam mantled corium particles has no effect on this approximate solution and the total heat loss from particles coincides with the heat absorbed in water. At the same time, the large-cell model includes the radiation heat transfer between the particles of different temperatures. In Lagrangian modeling of motion and cooling of an isothermal particle of radius a_i, the following energy equation is usually employed:

$$\frac{\rho c a_i}{3} \frac{\mathrm{d}T_i}{\mathrm{d}t} = -h(T_i - T_\mathrm{w}) - \psi_i \varepsilon_\mathrm{c} \sigma T^4_i \tag{30}$$

It is assumed here that the particle is totally opaque and optically gray ($\varepsilon_\mathrm{c} = \mathrm{const}$). Generally speaking, $\psi_i \neq 1$ and the value of ψ_i can be determined from the large-cell model. By using the same assumption on optical properties of corium particles, one can find:

$$\psi_i = 1 - \frac{\pi}{\sigma T^4_i} \int_{\lambda_1}^{\lambda_*} \frac{\alpha_\mathrm{c}}{\alpha_\lambda} \overline{B}_\lambda \,\mathrm{d}\lambda \tag{31}$$

One can see that $\psi_i < 1$ for all particles. To clarify the physical sense of coefficient ψ, consider the case of monodisperse corium particles when $\overline{B}_\lambda = B_\lambda(T)$ and Eq. (31) can be written in the form:

$$\psi = 1 - \zeta, \quad \zeta = \zeta_0 \int_{\lambda_1}^{\lambda_*} \frac{\alpha_\mathrm{c}}{\alpha_\lambda} B_\lambda(T) \,\mathrm{d}\lambda \Big/ \int_{\lambda_1}^{\lambda_*} B_\lambda(T) \,\mathrm{d}\lambda \tag{32}$$

where $\zeta_0(T)$ is the part of blackbody radiation at temperature T in the range of water semi-transparency:

$$\zeta_0(T) = \int_{\lambda_1}^{\lambda_*} B_\lambda(T) \,\mathrm{d}\lambda \Big/ \int_{\lambda_1}^{\lambda_2} B_\lambda(T) \,\mathrm{d}\lambda \tag{33}$$

Obviously, the coefficient ψ varies in the range between $1 - \zeta_0$ and 1, where the lower limit corresponds to high volume fraction of corium.

5. Comparison of diffusion and large-cell models for typical problem parameters

In this section, we consider a one-dimensional axisymmetric problem of radiation heat transfer in water containing polydisperse steam bubbles and steam mantled corium particles. In our example problem, we use the following similar profiles of the volume fractions of corium and steam:

$$f^\mathrm{c}_\mathrm{v}(r) = f^\mathrm{s}_\mathrm{v}(r) = f_{\mathrm{v}0}\varphi(r), \quad \varphi(r) = \frac{1}{1 + 5(r/R)^2} \tag{34}$$

The following fixed values of parameters are considered: $R = 0.5$ m, $f_{\mathrm{v}0} = 0.5\%$. The function $\varphi(r)$ and its "cell" approximation are shown in Fig. 4. The ordinates of the cell approximation for the number of cells $N = 10$ are calculated as follows:

$$\varphi_i = \frac{\int_{r_i}^{r_{i+1}} \varphi(r) r \,\mathrm{d}r}{\int_{r_i}^{r_{i+1}} r \,\mathrm{d}r} = \frac{1}{5(\bar{r}^2_{i+1} - \bar{r}^2_i)} \ln \frac{1 + 5\bar{r}^2_{i+1}}{1 + 5\bar{r}^2_i},$$

$$\bar{r}_i = \frac{r_i}{R} = \frac{i-1}{N} \quad i = 1, 2, \ldots, N+1 \tag{35}$$

The average radius of bubbles is assumed to be equal $a^\mathrm{b}_{32} = 3$ mm. All corium particles are assumed to be totally opaque. The emissivity of bulk corium was assumed to be independent of wavelength and temperature and equal to $\varepsilon_\mathrm{c} = 0.85$ [43]. Because of complexity of the general problem, two variants of the example problem are considered below: for monodisperse corium particles and model polydisperse corium characterized by different temperatures of small and large particles.

5.1. Monodisperse corium particles

Consider the case of monodisperse corium particles of radius $a_2 = 2.5$ mm and temperature $T = 3000$ K. The results of calculations based on P_1 approximation are

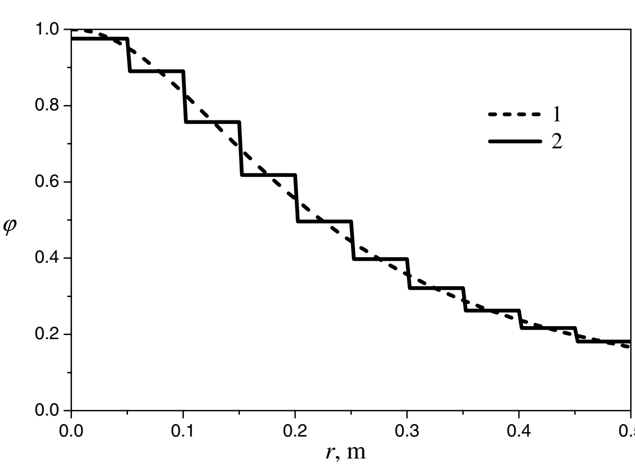

Fig. 4. Dimensionless profile of volume fraction of steam and corium considered in the model problem: 1 – smooth profile, 2 – step-wise approximation.

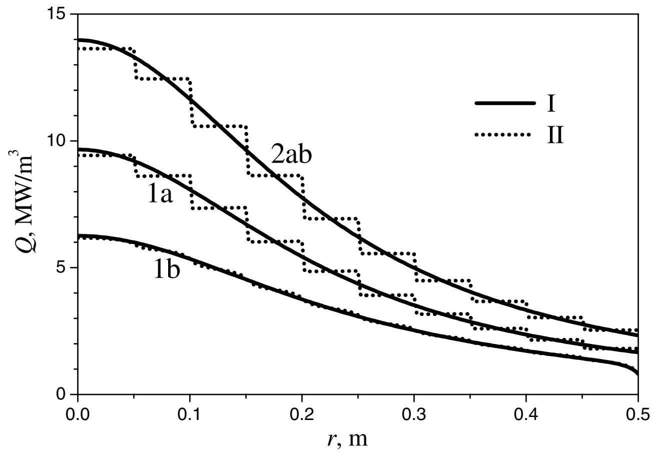

Fig. 5. Radiative heat loss from corium particles (a) and radiation power absorbed in water (b): 1 – in the range of water semi-transparency, 2 – in the range of water opacity; I – smooth profile, II – step-wise approximation.

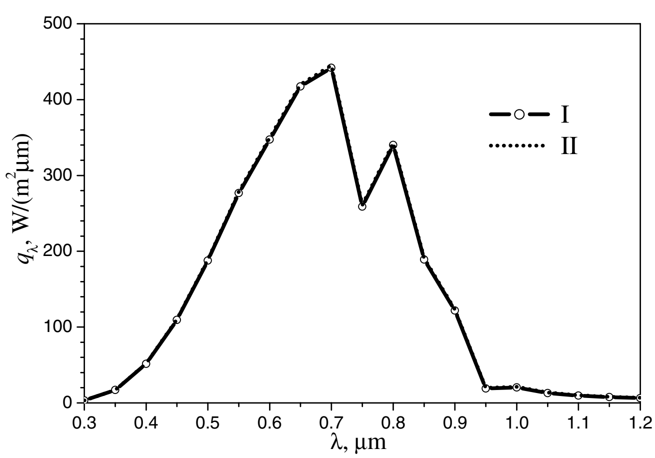

Fig. 6. Spectral radiative flux at the boundary of the computational region: calculations for smooth profile (I) and step-wise approximation (II) of the medium parameters.

presented in Figs. 5 and 6. One can see in Fig. 5 that there is a considerable difference between the radiation power emitted by corium particles in semi-transparency range and the power absorbed in water. It is explained by considerable radiative flux from the region in this spectral range (see Fig. 6). The difference between the calculations for smooth profile of the particle volume fraction and step-wise profile typical for cell approximation of the flow parameters is insignificant, especially for radiation power absorbed in water and spectral radiative flux at the region boundary. One can see in Fig. 6 that thermal radiation from the multiphase medium can be observed only in the visible range and the corresponding radiative heat loss is negligible in the medium heat balance.

It follows from Eq. (28) that large-cell model gives the only profile of radiation power. This profile is intermediate between the profiles obtained for corium and water in P_1

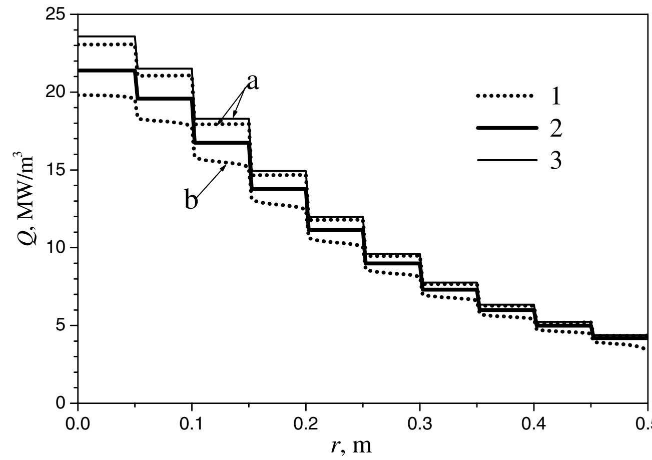

Fig. 7. Total radiative heat loss from corium particles (a) and radiation power absorbed in water (b): 1 – P_1 approximation, 2 – large-cell model, 3 – maximum estimate (36).

approximation. One can see in Fig. 7 that relative error of the large-cell model in total radiation power is not large (about 5–10%) because of decisive contribution of the opacity range. It is important that this error can be estimated by comparison of the large-cell solution with the upper limit of radiative heat loss from corium particles

$$P_{\mathrm{w}}^{\max} = \int_{\lambda_1}^{\lambda_2} p_\lambda \,\mathrm{d}\lambda \tag{36}$$

The latter statement is illustrated by curve $P_{\mathrm{w}}^{\max}(r)$ plotted in Fig. 7.

5.2. Polydisperse corium particles

For simplicity, the following two-mode size distribution of particles is considered:

$$F(a) = \xi\delta(a - a_1) + (1 - \xi)\delta(a - a_2) \tag{37}$$

with $a_1 = 0.5$ mm, $a_2 = 3$ mm, $T(a_1) = T_1 = 2000$ K, $T(a_2) = T_2 = 3000$ K. Obviously,

$$a_{ij} = \frac{\xi a_1^i + (1-\xi)a_2^i}{\xi a_1^j + (1-\xi)a_2^j} \tag{38}$$

Note that ξ is the relative number of small particles whereas the more representative volume fraction of these particles is given by $v = \xi a_1^3/a_{30}$. The effect of polydisperse corium particles can be analyzed on the basis of large-cell model. The local character of this model allows us to consider a single cell of the medium. One can write the following expressions for radiative cooling rate coefficients:

$$\psi_i(\xi) = 1 - \frac{\xi a_1^2 \zeta(T_1)T_1^4 + (1-\xi)a_2^2\zeta(T_2)T_2^4}{a_{20}T_i^4} \quad i = 1,2 \tag{39}$$

Note that calculations showed the predominant role of visible radiation in direct heat transfer between the particles of different temperatures. For this reason, it is sufficient

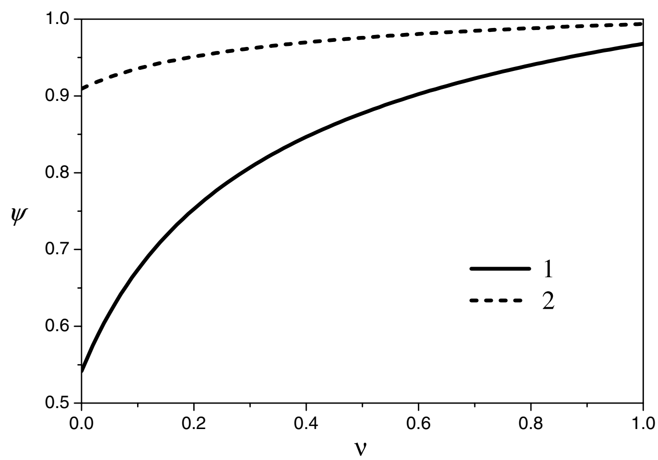

Fig. 8. The coefficients of radiative cooling rate for corium particles of two fractions as functions of relative volume fraction of small particles: $1 - \psi_1$, for small particles; $2 - \psi_2$, for large particles.

to use the "red" boundary of the visible spectral range $\lambda_i = 0.8$ μm instead of λ^* and the Wien law [43] in approximate calculations of function $\zeta(T)$. The results of calculations presented in Fig. 8 showed that thermal radiation from relatively hot large particles of corium in the visible spectral range can lead to significant decrease in the radiative cooling rate of small particles. This effect should be taken into account in calculations of the fuel–coolant interaction.

6. Conclusions

The visible and near-infrared radiative properties of water containing polydisperse steam bubbles and core melt particles are analyzed. Simple analytical expressions are derived for absorption and scattering characteristics of steam-mantled corium particles and steam bubbles in the range of water semi-transparency.

The alternative approximate models of radiation heat transfer in the range of water semi-transparency are suggested. A more general model is based on P_1 approximation for 3D radiative transfer in a refracting, absorbing, scattering, and emitting multi-temperature medium. For the same medium, a new approach based on radiation balance equations for a large computational cell of size about several centimeters is developed. This approach called large-cell model takes into account not only water heating but also radiation heat transfer between the particles of different temperatures. The latter is important for FCI problems. An applicability of the large-cell model is analyzed for example problem with physical parameters typical for FCI characterized by not too high volume fraction of steam. It is shown that error of the new model is usually less than about 5–10% both in radiation heat loss from corium and local radiation power absorbed in water.

The calculations based on the large-cell-model show a significant decrease in radiative cooling rate of small particles due to radiation of relatively hot large particles of corium in the visible spectral range. This effect should be taken into account in calculations of the fuel–coolant interaction.

The model developed can be easily generalized to the case of large volume fraction of steam when steam volumes contain polydisperse corium particles and water droplets. The resulting large-cell model can be considered for implementation in CFD codes oriented to FCI problems.

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