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An effect of turbulent clustering on scattering of microwave radiation by small particles in the atmosphere

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A coherent scattering of electromagnetic waves by clusters of inertial Rayleigh particles in atmospheric turbulence is considered. A preliminary estimate based on the Maxwell-Garnett theory and the Rayleigh approximation for single clusters demonstrates an importance of the coherent scattering contribution. It is confirmed by a general solution in a combination with theoretical estimates for the two-point probability density function for low-inertia spherical particles in isotropic turbulence. An approximate analytical expression for the coefficient characterizing effect of coherent scattering by the particle clusters is derived. The calculations for small Stokes numbers typical of water droplets in cumulus clouds yield an estimate of the coherent scattering effect on the microwave radar reflection. The model suggested allows solving the inverse problem to determine the pair correlation function for cloud particles. It is expected to be important for the investigations on particle–turbulence interaction in the atmosphere. The theoretical model developed is true not only in the limit of low-inertia particles and can be potentially used at arbitrary Stokes numbers in other applications.

1. Introduction

The understanding and adequate modeling of particle–turbulence interaction in the Earth atmosphere, especially in the atmospheric clouds, is very important for both cloud physics and global problems of meteorology. This interaction including turbulent clustering of inertial particles is a subject of numerous investigations during last decades [1]. The formation of clusters, that is, compact regions of preferential concentration surrounded by low-concentration zones, is one of the most challenging and complicated phenomena caused by the interaction of particles with fluid turbulent eddies. Clustering (or the accumulation effect) manifests itself as a tendency of heavy particles to avoid regions of high-vorticity and preferentially concentrate in regions of high strain rate [2]. The turbulent clustering is a result of fluctuations of inertial particle concentration due to compressibility of the velocity field of condensed phase. It may take place even in the case of incompressible carrier medium when there is a strong correlation between the flow vorticity field and the regions of preferential particle concentration [3,4]. The local accumulation of heavy particles is observed in low-vorticity regions under the action of centrifugal force and is mainly governed by small-scale turbulent structures. Therefore, the clustering of particles is most remarkable when the particle response time is comparable to the Kolmogorov time scale of fluid turbulence. For large Stokes numbers, the particle concentration distributions become defocused because high-inertia particles are weekly responsive to the small-scale vorticity of the fluid. In [5,6], the clustering phenomenon in homogeneous turbulence is treated as a result of a particle migration drift in the separation direction due to

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Nomenclature

a	particle radius
A	specific absorption coefficient
c	coefficient in Eq. (20)
C	cross-section
f_v	volume fraction
g	radial distribution function
I	radiation intensity
k	wave number
m	complex index of refraction
n_λ	index of refraction
n	number density of particles
N	number of particles
r	radial coordinate
R	cluster radius
Re	Reynolds number
s	independent variable in Eq. (16)
S	specific scattering coefficient
St	Stokes number
u	velocity
V	volume
x	diffraction parameter

Greek letters

α	polarizability
Δ	clustering coefficient
ε	kinetic-energy dissipation rate
k_λ	index of absorption
η	Kolmogorov's micro-scale of turbulence
θ	scattering angle
λ	wavelength
ν	kinematic viscosity
ρ	density
τ	characteristic time
χ	exponent in Eq. (20)

Subscripts

a	absorption
cl	cluster
cr	critical
p	particle
s	scattering
λ	spectral

the gradient of the radial relative fluctuating velocity. This drift leads to the collection of particle pairs at small separations and hence acts in perfect analogy to an additional attractive force. Thus, despite the stochastic nature of turbulence, the spatial distribution of inertial particles even in statistically homogeneous turbulent flow exhibits some regular trends.

Various criteria may be employed for specifying preferential particle concentration. Extensively employed way of measuring the accumulation effect (e.g., see [2,7–9]) is based on a deviation of the probability density of the number of particles located in fixed space cells from the Poisson law, which corresponds to the statistically independent random spatial distribution of particles. However, the most convenient way of quantifying the accumulation effect is using the radial distribution function (RDF) that is defined as the probability of observing a particle pair normalized by the corresponding value in a uniform suspension, because this quantity directly enters into the equations for the collision rate, the coagulation kernel, and so forth [10–16]. The same quantity is widely used in the statistical theory of liquids [17–19].

It is known that experimental data for radial distribution of molecules in liquids can be obtained from the measurements of X-ray scattering [17,18]. It is natural to assume that similar diagnostics can be developed to analyze a distribution of macroscopic particles in turbulent clusters. For atmospheric clouds containing water droplets of diameter $<100\,\mu$m, one can use the coherent Rayleigh scattering of microwave radiation. This general idea has been formulated in the paper by Kostinski and Jameson [20] as well as in the recent review by Shaw [1].

Of great interest is the derivation of analytical solutions to the problem of preferential concentration of low-inertia particles at as large Reynolds numbers as these can be achieved in atmospheric conditions, because to date it is not possible to reach the atmospheric Reynolds number in direct numerical simulation (DNS). In addition, the values of the Stokes number and the particle size related to the Kolmogorov length scale for typical atmospheric aerosols and rain droplets are less than those realizable in both direct simulations and laboratory experiments. It is well known that the feasibility of obtaining universal solutions to the problem of relative dispersion of particle pairs is associated with the phenomenon of intermittency caused by fluctuations of the turbulence dissipation rate [21–23]. The classical Kolmogorov similarity hypotheses for small-scale turbulence are justified when disregarding the effect of intermittency. These hypotheses establish the universality of small-scale turbulence, i.e., presume that the characteristics of turbulence in the viscous and inertial ranges at high Reynolds numbers are independent of large-scale vortex structures. Intermittency causes the features of small-scale turbulence to depend on the Reynolds number. Therefore, it is possible to derive universal (self-similar with regard to the Reynolds number) solutions only for not-too-high-inertia particles whose response times belong to the viscous or inertial interval, with the part played by intermittency being insignificant. The presence of such universality for the RDF is confirmed in DNS by Collins and Keswani [24], at least for particles with the response time belonging to the viscous interval.

Theoretical analysis of the turbulent clustering effect on propagation of microwave radiation in the atmosphere is of current interest now due to the new achievements in the technique of microwave remote sensing and to the progress in global geophysical research programs [25–28]. Note that the effect of turbulent clustering takes place not only for small water droplets in clouds [1,29] but also for large rain droplets [30].

The measurements of cloud reflectivity in the millimeter wavelength range are more sensitive to small cloud particles than the long-wave measurements. Therefore, the millimeter-range reflectivity is considered as an important source of information on water content in cumulus clouds [31,32]. The millimeter waves are absorbed in clear atmosphere but there are two important windows of transparency which are usually used for the measurements: the wavelength λ=8.6 and 3.2 mm [32]. Note that the millimeter-range measurements are not effective for warm clouds containing large and strongly scattering droplets of drizzle [33].

The measured reflection of microwave radar radiation from atmospheric clouds is usually explained by two mechanisms of scattering. The first one is an incoherent Rayleigh scattering by single particles. It is important in a short-wave range and decreases strongly with the wavelength. The second mechanism is a coherent scattering by spatial variations in the atmospheric index of refraction due to turbulent fluctuations of both temperature and humidity of air in the cloud. This coherent scattering is usually called the Bragg scattering or clear-air scattering and it is not related with water droplets. Spatial variations in the droplet volume fraction due to turbulent clustering can also contribute to the coherent scattering. But this effect was long thought to be insignificant because the relation between radar reflectivity of cumulus clouds and the liquid water content is usually well predicted by the model based on the Rayleigh scattering by single droplets. Kostinski and Jameson [20] were probably the first who paid the attention to the problem of using the coherent scattering by droplet clusters as a possible way to measure cloud texture.

The experimental data reported by Baker et al. [34] and Knight and Miller [35] for the X-band (λ=3.2 cm) and the S-band (λ=10 cm) reflectivity of developing small cumulus clouds and three-frequency radar measurements of a smoke plume produced by an intense industrial fire by Rogers and Brown [36] in the wavelength range from λ=3.2 to 32.8 cm have indicated a considerable discrepancy in the reflectivity spectral dependence from the theoretical predictions. According to [37], it can be explained by significant contribution of a coherent microwave scattering by clusters formed in turbulent transfer of the inertial particles. In the case of cumulus clouds, the coherent scattering by clusters of water droplets may be a predominant mechanism of scattering, especially in the centimeter spectral range [37].

An objective of the present paper is to estimate the effect of turbulent clustering of inertial particles on coherent scattering of the microwave radiation from clouds containing small water droplets. We consider the conditions realized in the cloudy atmosphere and use typical parameters of the continental cumulus clouds. At the same time, the general method developed in the paper is expected to be applicable to quite different objects such as organic particles suspended in the ocean or interstellar dust clouds in the Galaxy.

In our case, the predominant absorption of microwave radiation by water droplets (as compared with relatively small scattering) makes possible the use of single-scattering approach without an analysis of multiple scattering effects and solving the radiative transfer equation. The other simplification is a result of relatively small size of single droplets which are much smaller than the radiation wavelength. The above special features of the particular problem enable us to use much simpler theoretical approach that the numerical exact solutions of the Maxwell equations used by Mishchenko et al. [38] and not to solve the radiative transfer equation as was done recently by Loiko and Berdnik [39] for a layer of nonabsorbing optically soft particles. At the same time, we use the interference approximation similar to that employed by Loiko and Berdnik [39]. Note that we do not analyze the effects typical for a specific case of densely packed weakly absorbing particles of diffraction parameter in the Mie scattering region as was reported recently by Okada and Kokhanovsky [40].

The present paper is organized as follows. In Section 2, we give a preliminary physical estimate based on a very simplified model of homogeneous isolated Rayleigh clusters of single particles. This enables us to understand a possible nature of the substantial increase in scattering due to the coherent effects. In Section 3, which is the main part of the paper, a general formulation is considered and the realistic characteristics of turbulent clustering of low-inertia particles are used in the analysis.

2. Microwave absorption and scattering by water droplets and their clusters

A simple phenomenological model for interaction of external microwave radiation with a cloud of water droplets is considered in this section. This model is based on the radiation transfer theory and the corresponding presentation of the cloud as an optically homogeneous medium characterized by absorption and scattering coefficients [41,42]. A physical estimate of these coefficients for single Rayleigh particles and their clusters is given below.

The clusters are imagined as small spherical clouds of monodisperse particles. In contrast to [43,44], we do not consider the agglomerates or the clusters of densely packed particles. It is also important that the distances between the neighboring clusters are assumed to be much greater that the cluster size. The latter enables us to focus on radiative properties of single clusters. For simplicity, we assume that the volume fraction of particles is constant across the cluster. Moreover, it is assumed that the diameter of cluster is much less than the radiation wavelength. In this case, the cluster can be treated as a homogeneous Rayleigh particle with an equivalent complex index of refraction determined by the Maxwell-Garnett theory [45–47]. Maxwell-Garnett has considered a "substance" formed by randomly placed spherical particles with the polarizability determined by the

Lorentz formula:

$$\alpha = \frac{m^2-1}{m^2+2}a^3 \tag{1}$$

where α is the particle radius and m is the complex index of refraction of the particle substance. For clusters of size much greater than the distances between neighboring particles, one can employ the Lorentz–Lorenz equation for the equivalent complex index of refraction of the "cluster substance":

$$m_*^2 = 1 + 4\pi n_{cl}\alpha/(1 + 4\pi n_{cl}\alpha/3) \tag{2}$$

where n_{cl} is the number density of particles in the cluster. It is assumed that particles are suspended in air, i.e. in a medium with complex index of refraction $m_e = 1$. Obviously, Eq. (2) can be written in the form:

$$\frac{m_*^2-1}{m_*^2+2} = \frac{f_v\overline{\alpha}}{1+2f_v\overline{\alpha}}, \quad \overline{\alpha} = \frac{\alpha}{a^3} = \frac{m^2-1}{m^2+2} \tag{3}$$

where $f_v \equiv n_{cl}V$ is the volume fraction of particles in the cluster, $V \equiv 4\pi a^3/3$ is the particle volume. In the case of not too high volume fraction of particles ($f_v \ll 1$), Eq. (3) is reduced to

$$\frac{m_*^2-1}{m_*^2+2} = f_v\frac{m^2-1}{m^2+2} \tag{4}$$

Let us compare the optical properties of a cluster with the properties of a rarefied cloud containing the same particles. The absorption and scattering cross-sections of single homogeneous spherical particles in the Rayleigh region when

$$x = ka \equiv \frac{2\pi a}{\lambda} \ll 1, |m|x \ll 1 \tag{5}$$

are determined as follows:

$$C_a = \frac{6\pi V}{\lambda}\mathrm{Im}\left(\frac{1-m^2}{m^2+2}\right), \quad C_s = \frac{24\pi^3 V^2}{\lambda^4}\left|\frac{m^2-1}{m^2+2}\right|^2 \tag{6}$$

Similar equations for a homogeneous Rayleigh cluster of radius R are

$$C_{a,cl} = \frac{6\pi V_{cl}}{\lambda}\mathrm{Im}\left(\frac{1-m_*^2}{m_*^2+2}\right), \quad C_{s,cl} = \frac{24\pi^3 V_{cl}^2}{\lambda^4}\left|\frac{m_*^2-1}{m_*^2+2}\right|^2, \quad V_{cl} = \frac{4}{3}\pi R^3 \tag{7}$$

Having in mind Eqs. (4) and (6) we can write Eqs. (7) in the form

$$C_{a,cl} = C_a N_{cl}, \quad C_{s,cl} = C_s N_{cl}^2 \tag{8}$$

where $V_\Sigma = f_v V_{cl}$ is the total volume of particles, $N_{cl} = V_\Sigma/V$ is the number of particles in the cluster. It is of interest to compare the values obtained with the values of similar quantities for a cloud of particles which are not collected in the cluster:

$$C_{a,\Sigma} = C_a N_{cl}, \quad C_{s,\Sigma} = C_s N_{cl} \tag{9}$$

One can see from Eqs. (8) and (9) that the total absorption remains the same; whereas the scattering increases by a factor of N_{cl} due to the cluster formation: $C_{s,cl} = C_{s,\Sigma}N_{cl}$.

At not too high volume fraction of particles, the equivalent complex index of refraction of the "cluster substance" satisfies the known condition of an "optically soft" particle, that enables us to remove the Rayleigh limitation of $x_{cl} = kR \ll 1$ and consider much bigger clusters, which satisfy the conditions of the Rayleigh–-Gans scattering [48,49]:

$$|m_* - 1| \ll 1, \quad 2x_{cl}|m_* - 1| \ll 1 \tag{10}$$

In the Rayleigh–Gans region, every small element of the cluster scatters the radiation as a Rayleigh particle independently on other volume elements. The waves scattered in a given direction interfere due to the different spatial positions of the volume elements. For a homogeneous spherical cluster, one can use the following relations for the equivalent complex index of refraction and for the polarizability:

$$m_* = 1 + 1.5f_v\frac{m^2-1}{m^2+2}, \quad \alpha_{cl} = \frac{2}{3}(m_* - 1)R^3 \tag{11}$$

The Rayleigh–Gans approximation allows also taking into account the radial profiles of volume fraction of particles in the spherical clusters. An analytical solution for this problem has been derived by van de Hulst [48]. In the case of small clusters, the scattering cross-section of a radially inhomogeneous cluster can be calculated by using the Rayleigh formula:

$$C_{s,cl} = \frac{8\pi}{3}k^4\alpha_{cl}^2 \tag{12}$$

but with the following integral polarizability of the cluster:

$$\alpha_{cl} = 2\int_0^R [m_*(r) - 1]r^2\,dr = 3\frac{m^2-1}{m^2+2}\int_0^R f_v(r)r^2\,dr \tag{13}$$

The final expression looks similar to that for homogeneous clusters:

$$C_{s,cl} = C_{s,\Sigma}N_{cl}, N_{cl} = \frac{3}{a^3}\int_0^R f_v(r)r^2\,dr \tag{14}$$

It is important that the formation of clusters does not influence on the radiation absorption but leads to the significant increase in scattering. The latter effect is directly proportional to the number of particles in the cluster. It should be emphasized that the above estimates of absorption and scattering of long-wave radiation by spherical clusters are correct when (1) the number of particles in the cluster is large and (2) the volume fraction of particles in the cluster is not too large. It goes without saying that the real clusters are not spherical and the above estimates should be corrected. The radiation scattering by randomly oriented elongated clusters is expected to be greater than the scattering by hypothetical spherical clusters [50,51]. The Rayleigh–Gans approximation enables one to consider the clusters of arbitrary shape but the present analysis is limited to the case of spherical clusters.

Let us consider the specific spectral coefficients of absorption and scattering for a cloud of small water droplets: $A_\lambda = \alpha_\lambda/f_v = C_a/V$ and $S_\lambda = \sigma_\lambda/f_v = C_s/V$, where α_λ and σ_λ are the ordinary absorption and scattering coefficients. The calculations were performed by using the known data for the spectral optical constants of water n_λ and κ_λ ($m_\lambda = n_\lambda - i\kappa_\lambda$) [52,53]

(see Fig. 1). The results presented in Fig. 2 indicate that the scattering is relatively small and decreases strongly with the wavelength. It means that the microwave observations of turbulent clustering in cumulus clouds are expected to be reliable only in the case of significant contribution of the coherent scattering by droplet clusters.

3. Theoretical model for single coherent scattering by Rayleigh particles

As in the previous section, we assume that the Rayleigh conditions (5) are satisfied for every particle. But the further analysis will not be based on scattering properties of the conventional single clusters. Instead, we employ the general problem statement for the interference of electromagnetic waves scattered by arbitrary positioned particles and use the so-called independent scattering hypothesis when each particle is assumed to scatter the radiation in exactly the same manner as if all the other particles did not exist. The weak scattering (as that by atmospheric clouds in the microwave) enables one to employ single-scattering approximation when the amplitude of scattered electromagnetic wave is determined simply as a sum of the amplitudes of the waves scattered by the particles. It is

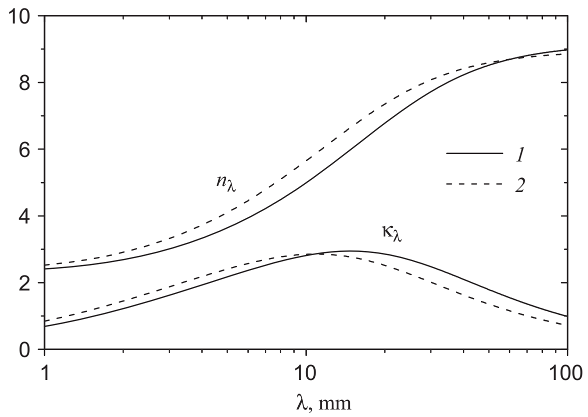

Fig. 1. Optical constants of water in the microwave range at temperatures 283 K (1) and 293 K (2).

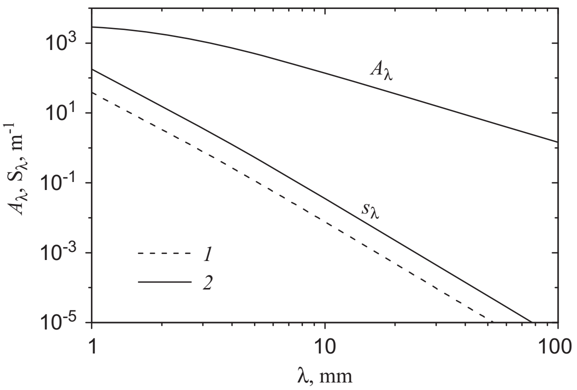

Fig. 2. Specific coefficients of absorption and scattering for water droplets of radius 30 μm (1) and 50 μm (2) at temperature 283 K.

known that the above conditions are satisfied for X-ray scattering from liquids [17–19] and for scattering of radio-waves by discrete macroscopic particles in a dielectric ambient medium [54,55].

Assuming that the observation point is in the Fraunhofer zone, i.e. far enough from the scattering volume, one can derive the following equation for the average intensity of scattered radiation:

$$I(\theta) = I_0(\theta)(1+\Delta) \tag{15}$$

where I is the intensity of scattered radiation, I_0 is the intensity of scattered radiation calculated by neglecting the coherent scattering (i.e. without taking into account the interference of radiation scattered by particles with correlated spatial coordinates), θ is the angle of scattering, and Δ is the coefficient of clustering. The value of Δ does not depend on radiative properties of particles and it is determined by the following equation [17,18]:

$$\Delta(s) = \frac{4\pi n}{s}\int_0^\infty [g(r)-1]\sin(sr)r\,dr, \quad s = 2k\sin(\theta/2) \tag{16}$$

where n is the number density of particles, $g(r)$ is the radial distribution function (RDF), and r is the distance between particles. One can see that coefficient Δ can be treated as the Fourier sinus-transform of $4\pi n[g(r)-1]r/s$. Taking the Fourier inversion formula we obtain

$$g(r) = 1 + \frac{1}{2\pi^2 nr}\int_0^\infty \Delta(s)\sin(sr)s\,ds \tag{17}$$

Eq. (17) can be used to identify the radial distribution function from the experimental data for the coefficient of clustering.

It is interesting to consider the conditions when Eqs. (15) and (16) are reduced to the simple relation $C_{s,cl} = C_{s,\Sigma}N_{cl}$ for effect of clustering on coefficient of scattering. First of all, one should use the assumption of Rayleigh scattering by clusters. It makes Eq. (16) much simpler:

$$\Delta = 4\pi n\int_0^\infty [g(r)-1]r^2\,dr \tag{18}$$

If we integrate only over the volume of cluster and assume that volume fraction of particles in the cluster is constant and much greater than the average value in the medium ($g \gg 1$), the result is $I = I_0 N_{cl}$, i.e. exactly the same as that derived in the previous section for the homogeneous Rayleigh clusters.

Coming back to the general Eq. (16) we can estimate the clustering coefficient for small low-inertial particles of radius a much less than the Kolmogorov micro-scale of turbulence $\eta \equiv (\nu^3/\varepsilon)^{1/4}$, where ν is the kinematic viscosity of the carrier medium and ε is the kinetic-energy dissipation rate. Note that this condition of low inertia of particles is satisfied for the majority of atmospheric problems due to a large value of η typical of the atmospheric turbulence. According to [5,6], the function $g(r)$ can be determined by employing a statistical model based on the kinetic equation for the probability density function for relative velocity of two particles in a homogeneous isotropic turbulence. This model was modified in [56] for more accurate description of clustering of low-inertia particles. The modification takes into account a difference between the time scales of strain and rotation rate correlations. In the case of $a \ll \eta$ and negligible hydrodynamic interaction of particles, the function g depends only on dimensionless distance between the particles $\overline{r} \equiv r/\eta$, the Stokes number is $\mathrm{St} \equiv \tau_p/\tau_K$, and the Reynolds number, $\mathrm{Re} \equiv (15u'^4/\varepsilon\nu)^{1/2}$, is determined for the Taylor spatial scale of turbulence. Remember that τ_p is the dynamic relaxation time of the particle, $\tau_K \equiv (\nu/\varepsilon)^{1/2}$ is the Kolmogorov time micro-scale of turbulence, and u' is the mean-square pulsating velocity of the carrier medium. The typical radial distribution functions calculated using the model of [56] are shown in Fig. 3. One can see that the effect of clustering characterized by a maximal deviation of function g from unity at small values of $\overline{r}$ is more pronounced at $\mathrm{St} \approx 1$. This result is physically clear because very small particles ($\mathrm{St} \ll 1$) follow the turbulent medium and the large particles ($\mathrm{St} \gg 1$) are weakly sensitive to the turbulent fluctuations of the carrier fluid. Fig. 4 demonstrates a good agreement of the RDF predicted for low-inertia particles with measurements by Salazar et al. [57] obtained using digital holographic imaging technique. It is also seen that the RDF of low-inertia particles has a singularity at $r=0$. At relatively large Stokes numbers, this singularity disappears and g takes a constant value at $r=0$. The unrestricted growth of the radial distribution function with the separation distance tending to zero (i.e., a singularity of the RDF at the origin) was previously found for low-inertia zero-size particles by both analytical [5,56,58,59] and numerical [10,24,57,59,60] means. It should be also noted that the level of preferential concentration (clustering) can be less for cloud droplets that it is predicted by the theory. This is mainly caused by gravitational settling that reduces the interaction time of droplets with fluid turbulent eddies. One can overcome this deficiency by taking into account the effect of gravity on the RDF. However, it is beyond the scope of the present paper and we consider this analysis as a subject of a separate study.

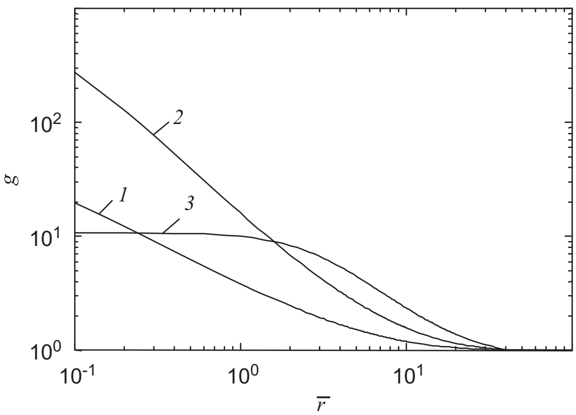

Fig. 3. Radial distribution function at Re=75: 1—St=0.5, 2—St=1, 3—St=2.

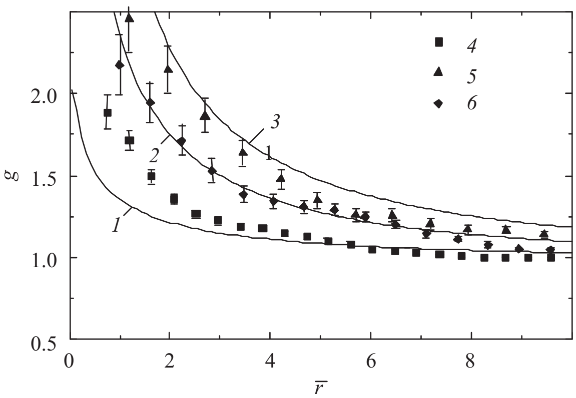

Fig. 4. Radial distribution function of low-inertia particles: 1–3—theoretical predictions; 4–6—experiments [57]; 1, 4—St=0.21, Re=108; 2, 5—St=0.4, Re=134; 3, 6—St=0.6, Re=147.

In the case of the Stokes flow around a particle, the time of the particle dynamic relaxation and the Stokes number are

$$\tau_p = \frac{2}{9}\frac{\rho_p}{\rho}\frac{a^2}{\nu}, \quad \mathrm{St} = \frac{2}{9}\frac{\rho_p}{\rho}\overline{a}^2, \quad \overline{a} = a/\eta \tag{19}$$

where ρ_p and ρ are the densities of the particle substance and the carrier medium, respectively. Remember that the Stokes flow takes place at small values of the particle Reynolds number defined as $\mathrm{Re}_p = 2a|u-u_p|/\nu \ll 1$. This condition is satisfied in many important problems including the motion of atmospheric aerosols and water droplets in clouds [1].

Let us estimate the conditions of clustering of water droplets in atmospheric clouds. The Reynolds number may reach the value 10^4 in cumulus clouds, whereas the dimensionless particle radius $\overline{a}$ is not >0.01 and the Stokes number St is <0.1 (according to [1] the range of $\mathrm{St} \sim 10^{-3} - 10^{-1}$ is typical of cumulus clouds). It was found in paper [56] that the following analytical solution takes place for low-inertial aerosol particles:

$$g = c/\overline{r}^{\chi} \tag{20}$$

where the coefficient c and the exponent χ are approximated as

$$c = 1 + 12\,\mathrm{St}^2 \tag{21}$$

$$\chi = 6\,\mathrm{St}^2 - 10.4\,\mathrm{St}^3 + 7\,\mathrm{St}^4 \tag{22}$$

Expressions (20)–(22) are valid for $\mathrm{St}<0.6$ and $\mathrm{Re}>30$. Solution (20) confirms the singularity of the RDF at $\overline{r} \to 0$. Fig. 5 compares the exponent χ with DNS data of [59] at Re=47.1. It is clear that the predictions of χ are in excellent agreement with the DNS.

Strictly speaking, Eq. (20) is correct at $\overline{r} \le 1$, but it is also used at larger values of $\overline{r}$ for qualitative estimating the clustering coefficient. Moreover, it is obvious that solution (20) is not valid for finite-size particles when $r<a$. However, the contribution of the integration interval $0<r<a$ to (16) is vanishingly small when $\overline{a} \ll 1$. Thus,

having substituted Eq. (20) in (16) we obtain:

$$\Delta = \frac{4\pi\overline{n}}{\overline{s}} \int_0^{\overline{r}_{cl}} \left(\frac{c}{\overline{r}^{\chi}} - 1 \right) \sin(\overline{s}\overline{r}) \overline{r}\, d\overline{r}, \quad \overline{s} = s\eta, \quad \overline{n} = n\eta^3 \tag{23}$$

The upper limit of integration in Eq. (23) is equal to the conventional cluster radius: $\overline{r}_{cl} = c^{1/\chi}$. At small values of $\overline{s}$, this equation can be written as the following convergent series:

$$\Delta = 4\pi\overline{n}\chi \sum_{j=0}^{\infty} \frac{(-1)^j \overline{s}^{2j} c^{(2j+3)/\chi}}{\Gamma(2j+2)(2j+3)(2j+3-\chi)} \tag{24}$$

where $\Gamma(\xi)$ is the gamma-function. In the case of $\mathrm{St} \ll 1$, Eq. (24) is reduced to

$$\Delta = 24\pi\overline{n}\,\mathrm{St}^2 \sum_{j=0}^{\infty} \frac{(-1)^j \overline{s}^{2j} e^{4j+6}}{\Gamma(2j+2)(2j+3)^2} \tag{25}$$

The first term of this series is responsible for the major contribution to the clustering coefficient:

$$\Delta = \frac{8\pi e^6}{3} \overline{n}\,\mathrm{St}^2 \tag{26}$$

Eq. (26) enables us to obtain an estimate of Δ in the most important range of the problem parameters. Note that coefficient of clustering is directly proportional to St^2 in the limit of small Stokes number. The values of $\Delta/\overline{n}$ calculated by Eqs. (23) and (26) are compared in Fig. 6. One can see that Eq. (26) yields a good estimate of the clustering coefficient when $\overline{s} < 0.5$ and $\mathrm{St} \leq 0.05$, i.e. in the realistic ranges of these parameters in microwave remote sensing of cumulus clouds. Note that we have $\overline{n} \approx 1$ at $n \approx 1\,\mathrm{mm}^{-3}$ and $\eta \approx 1$ mm, so that the values of $\Delta/\overline{n}$ in Fig. 5 can be treated as the absolute values of the clustering coefficient Δ. It is important that the value of Δ appears to be directly proportional to a^4, i.e. the clustering coefficient is very sensitive to the droplet size.

The above estimates indicate that the effect of turbulent clustering of small water droplets in atmospheric clouds may lead to a considerable increase in the microwave radiation scattering. But one should remember that it is a minimal estimate. A complex elongated shape

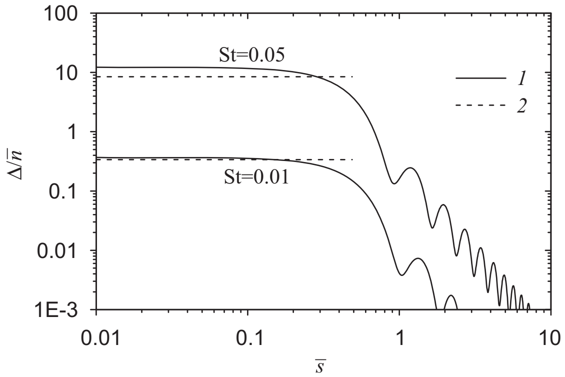

Fig. 6. Clustering coefficient for water droplets in the atmosphere: 1—numerical integration of (23) and 2—approximation (26).

of the real clusters at not-too-small Stokes number [1,29,61,62] is expected to provide more strong effect of the clustering on microwave scattering. It means that the microwave reflectivity data from radar measurements should be considered as a potential source of important information on turbulent clustering of water droplets in the clouds.

4. Conclusions

A coherent scattering of electromagnetic waves by clusters of inertial Rayleigh particles in the atmospheric turbulence is considered. An analysis based on the known Rayleigh solutions for small spherical particles and the Maxwell-Garnett theory for their clusters showed that the clustering of cloud droplets does not affect absorption but may increase significantly the radiation scattering in the microwave spectral range. This analysis is completed on the basis of the recent developments in theoretical estimating the two-point probability density function for low-inertia spherical particles in the isotropic turbulence. An approximate analytical dependence for the coefficient characterizing the effect of coherent scattering by the Rayleigh particle clusters is obtained. The calculations for small Stokes numbers typical of water droplets in cumulus clouds yield an estimate of the coherent scattering contribution to the microwave radar reflection. This value appears to be significant: the clustering coefficient is directly proportional to the fourth power of the droplet size. The real effect may be even greater due to the presence of randomly oriented elongated clusters.

The general expression for the coherent Rayleigh scattering by clusters is similar to that for the X-ray scattering from liquids. It allows solving the inverse problem to determine the pair correlation function for cloud particles. The latter is expected to be very important for the investigations on inertial particle–turbulence interaction in the atmosphere. The theoretical model developed is true not only in the limit of low-inertia

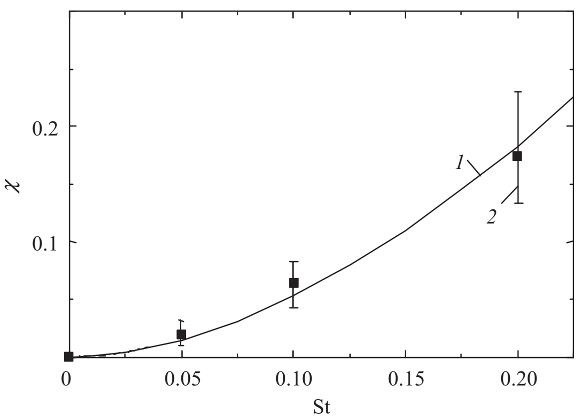

Fig. 5. Effect of Stokes number on the exponent χ: 1—approximation (22) and 2—DNS by Chun et al. [59].

particles and can be potentially used at arbitrary Stokes numbers in other applications.

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References

[1] Shaw RA. Particle–turbulence interactions in atmospheric clouds. Ann Rev Fluid Mech 2003;35:183–227.
[2] Squires KD, Eaton JK. Preferential concentration of particles by turbulence. Phys Fluids A 1991;3:1169–78.
[3] Maxey MR. The gravitational settling of aerosol particles in homogeneous turbulence and random flow fields. J Fluid Mech 1987;174:441–65.
[4] Elperin T, Kleeorin N, Rogachevskii I. Self-excitation of fluctuations of inertial particle concentration in turbulent fluid flow. Phys Rev Lett 1996;77:5373–6.
[5] Zaichik LI, Alipchenkov VM. Pair dispersion and preferential concentration of particles in isotropic turbulence. Phys Fluids 2003;15:1776–87.
[6] Zaichik LI, Alipchenkov VM, Sinaiski EG. Particles in turbulent flows. Weinheim: Wiley-VCH; 2008.
[7] Hogan RC, Cuzzi JN. Stokes and Reynolds number dependence of preferential particle concentration in simulated three-dimensional turbulence. Phys Fluids 2001;13:2938–45.
[8] Kostinski AB, Shaw RA. Scale-dependent droplet clustering in turbulent clouds. J Fluid Mech 2001;434:389–98.
[9] Sigurgeirsson H, Stuart AM. A model for preferential concentration. Phys Fluids 2002;14:4352–61.
[10] Reade WC, Collins LR. Effect of preferential concentration on turbulent collision rates. Phys Fluids 2000;12:2530–40.
[11] Wang L-P, Wexler AS, Zhou Y. Statistical mechanical description and modeling of turbulent collision of inertial particles. J Fluid Mech 2000;415:117–53.
[12] Zhou Y, Wexler AS, Wang L-P. Modelling turbulent collision of bidisperse inertial particles. J Fluid Mech 2001;433:77–104.
[13] Zaichik LI, Simonin O, Alipchenkov VM. Two statistical models for predicting collision rates of inertial particles in isotropic turbulence. Phys Fluids 2003;15:2995–3005.
[14] Bec J, Celani A, Cencini M, Musacchio S. Clustering and collisions of heavy particles in random smooth flows. Phys Fluids 2005;17:073301 [paper].
[15] Chun J, Koch DL. Coagulation of monodisperse aerosol particles by isotropic turbulence. Phys Fluids 2005;17:027102 [paper].
[16] Zaichik LI, Simonin O, Alipchenkov VM. Collision rates of bidisperse inertial particles in isotropic turbulence. Phys Fluids 2006;18:035110 [paper].
[17] Fisher IZ. Statistical theory of liquids. Chicago: University of Chicago Press; 1964.
[18] Hirschfelder JO, Curtiss CF, Bird RR. The molecular theory of gases and liquids. New York: Wiley; 1964.
[19] Kirkwood JG. Theory of liquids. New York: Gordon and Breach; 1968.
[20] Kostinski AB, Jameson AR. On the spatial distribution of cloud particles. J Atmos Sci 2000;57:901–15.
[21] Monin AS, Yaglom AM. Statistical fluid mechanics, mechanics of turbulence, vol. 2. Cambridge: MIT Press; 1975 [also Dover Publ., 2007].
[22] Frisch U. Turbulence. Cambridge: Cambridge University Press; 1995.
[23] Pope SB. Turbulent flows. Cambridge: Cambridge University Press; 2000.
[24] Collins LR, Keswani A. Reynolds number scaling of particle clustering in turbulent aerosols. New J Phys 2004;6:1–17.
[25] Janssen MA, editor. Atmospheric remote sensing by microwave radiometry. New York: Wiley; 1993.
[26] Simmer C. Contribution of microwave remote sensing from satellites to studies on the Earth energy budget and hydrological cycle. Adv Space Res 1999;24:897–905.
[27] Sharkov EA. Passive microwave remote sensing of the earth, physical foundations. Chichester, UK: Praxis; 2003.
[28] Woodhouse IH. Introduction to microwave remote sensing. New York: CRC Press; 2004.
[29] Khain A, Pinsky M, Elperin T, Kleeorin N, Rogachevskii I, Kostinski A. Critical comments to results of investigations of drop collisions in turbulent clouds. Atmos Res 2007;86:1–20.
[30] Jameson AR, Kostinski AB. Fluctuation properties of precipitation. Part IV: Observations of hyperfine clustering and drop size distribution structures in three-dimensional rain. J Atmos Sci 2000;57:373–88.
[31] Kropfli RA, Matrosov SY, Uttal T, Ok BW, Frisch AS, Clark, KA, et al. Cloud physics studies with 8 mm wavelength radar. Atmos Res 1995;35:299–313.
[32] Manheimer WM, Fliflet AW, Linde GJ, Cheung WJ, Gregens-Hansen V, Ngo, MT, et al. The structure of turbulence in clouds measured by a high power 94 GHz radar. Phys Plasmas 2004;11: 2852–2856.
[33] Pujol O, Georgis J-F, Sauvageot H. Influence of drizzle on Z–M relationships in warm clouds. Atmos Res 2007;86:297–314.
[34] Baker BA, Brenguier J-L, Cooper W. Unknown source of reflectivity from small cumulus clouds. In: Proceedings of American Meteorological Society Conference on Cloud Physics. Everett, WA, 1998. p. 148–51.
[35] Knight CA, Miller LJ. Early radar echoes from small, warm cumulus: Bragg and hydrometeor scattering. J Atmos Sci 1998; 55:2974–92.
[36] Rogers RR, Brown WOJ. Radar observations of a major industrial fire. Bull Am Meteor Soc 1997;78:803–14.
[37] Erkelens JS, Venema VKC, Russchenberg HWJ, Ligthart LP. Coherent scattering of microwaves by particles: evidence from clouds and smoke. J Atmos Sci 2001;58:1091–102.
[38] Mishchenko MI, Liu L, Mackowski DW, Cairns B, Videen G. Multiple scattering by random particulate media: exact 3D results. Opt Express 2007;15:2822–36.
[39] Loiko VA, Berdnik VV. Scattering by a plane-parallel layer with high concentration of optically soft particles. JQSRT 2009;110:1502–10.
[40] Okada Y, Kokhanovsky AA. Light scattering and absorption by densely packed groups of spherical particles. JQSRT 2009;110:902–17.
[41] Mishchenko MI, Travis LD, Lacis AA. Multiple scattering of light by particles: radiative transfer and coherent backscattering. New York: Cambridge University Press; 2006.
[42] Mishchenko MI. Multiple scattering, radiative transfer, and weak localization in discrete random media: unified microphysical approach. Rev Geophys 2008;46:RG2003 [paper].
[43] Khlebtsov NG. Spectroturbidimetry of fractal clusters: test of density correlation function cutoff. Appl Opt 1996;35:4261–70.
[44] Gruy F. Light-scattering cross section as a function of pair distribution density. JQSRT 2009;110:240–6.
[45] Maxwell-Garnett JC. Colours in metal glasses and in metallic films. Philos Trans R Soc Lond A 1904;203:385–420.
[46] Rozenberg GV. Optics of thin coatings. Moscow: Fizmatgiz; 1958 [in Russian].
[47] Choy TC. Effective medium theory: principles and applications. New York: Oxford University Press; 1999.
[48] Van de Hulst HC. Light scattering by small particles. New York: Wiley; 1957.
[49] Bohren CF, Huffman DR. Absorption and scattering of light by small particles. New York: Wiley; 1983.
[50] Dombrovsky LA. Radiation heat transfer in disperse systems. New York: Begell House; 1996.
[51] Dombrovsky LA. Radiative properties of particles and fibers. ThermalHUB publication, 2008. ⟨http://thermalhub.org/contributors/1050⟩.
[52] Ray PS. Broadband complex refractive indices of ice and water. Appl Opt 1972;11:1836–44.
[53] Zolotarev VM, Dyomin AV. Optical constants of water in wide wavelength range 0.1 Å–1 m. Opt Spectrosc 1977;43:271–9.
[54] Rytov SM, Kravtsov YuA, Tatarskii VI. Introduction to statistical radiophysics. Part II: stochastic fields. Moscow: Nauka; 1978 [in Russian].
[55] Apresyan LA, Kravtsov YuA. Radiation transfer: statistical and wave aspects. Singapore: Gordon and Breach; 1996.
[56] Zaichik LI, Alipchenkov VM. Refinement of the probability density function model for preferential concentration of aerosol particles in isotropic turbulence. Phys Fluids 2007;19:113308 [paper].
[57] Salazar JPLC, de Jong J, Cao L, Woodward SH, Meng H, Collins LR. Experimental and numerical investigation of inertial particle clustering in isotropic turbulence. J Fluid Mech 2008;600:245–56.
[58] Falkovich G, Fouxon A, Stepanov MG. Acceleration of rain initiation by cloud turbulence. Nature 2002;419:151–4.

[59] Chun J, Koch DL, Rani SL, Ahluwalia A, Collins LR. Clustering of aerosol particles in isotropic turbulence. J Fluid Mech 2005;536: 219–251.
[60] Falkovich G, Pumir A. Intermittent distribution of heavy particles in turbulent flow. Phys Fluids 2004;16:47–50.
[61] Tsuji T, Ito A, Tanaka T. Multi-scale structure of clustering particles. Powder Technol 2008;179:115–25.
[62] Calzavarini E, Kerscher M, Lohse D, Toschi F. Dimensionality and morphology of particle and bubble clusters in turbulent flow. J Fluid Mech 2008;607:13–24.